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ОБЧИСЛЕННЯ МОВОЮ C # СПЕЦІАЛЬНИХ ФУНКЦІЙ \( \text{igam}(a, x) \) ТА \( \text{igamc}(a, x) \)

В СТАТИСТИЧНOMУ МОДЕЛЮВАННІ

Запропоновано компактний варіант обчислення на мові C# таких специальних функцій як додаткова гама-функція, неповна додаткова гама-функція, натуральний логарифм від гама-функції та квантилі розподілення хі-квадрат, що використовуються під час вирішення задач статистичного моделювання.

Ключові слова: спеціальні функції, обчислювальні алгоритми, статистичне моделювання.

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CALCULATION IN C # SPECIAL FUNCTIONS \( \text{igam}(a, x) \) AND \( \text{igamc}(a, x) \)

FOR THE NEEDS OF STATISTICAL MODELING

A compact version of the calculations on the C # language such special functions such as: additional gamma function, incomplete gamma function additional, natural logarithm of the gamma function and the quantile of chi-square distribution used when solving problems of statistical modeling.

Keywords: special functions, numerical algorithms, statistical modeling.

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METHOD FOR ISAR IMAGING OBJECTS WITH 3D ROTATIONAL MOTION

The influence of random components of spatial target movement on phase dependencies of signals from different scatterers is analyzed. Imaging method, which along with the regular trajectory component of target movement takes into account the random spatial components of rotational motion, is developed. These components are measured by phase changes of bright points at radar image sequence obtained by the discrete Fourier transform, coordinates of points being estimated using parametric spectral analysis methods.

Keywords: radar high range resolution profile, ISAR imaging, cross-range, radar image, 3D rotational motion.

Introduction

One of the ways of increasing the reliability of object recognition is to find and use highly informative signal features. Employment of wideband probing signals by monostatic active radar permits to resolve the elements of targets along the line-of-sight (LOS) and observe their high range resolution profiles (HRRP). The so called cross-range profiles are similar to HRRP but unlike the latter they show the location of the radar scatterers projected onto the line perpendicular to the LOS. Such profiles may actually be considered as one-dimensional radar images of targets.

However if an aerial target is observed from its side (aspect angles of 70º ... 110º), its HRRP becomes uninformative because fuselage obstructs the view of some its elements [1-3]. Additionally, the jet-engine modulation signatures are almost undetectable [4, 5]. In such circumstances, the interest shifts to the two-dimensional (2D) radar images, for which the side observation aspects are the most favorable [1]. In works by H. Safronov [6], J. Zinoviev and A. Pasmurov [7, 8], D.R. Wehner [9], A. Rihaczek and S. Hershkowitz [10], B. Steinberg [11], M. Prickett and C. Chen [12], V. Chen and H. Ling [13], V. Chen and M. Martorella [14] and by others the theoretical framework and examples of inverse synthetics aperture radar (ISAR) imaging of aerial objects have been given, including data on field experiments [15-17 and other]. Examples of ISAR imaging by Beijing Institute of Radio Measurement in C-band (4-8 GHz) and by Japanese researchers for radar operation frequency of 9.65 GHz were presented in [15, 16]. They illustrate a possibility of ISAR images being successfully obtained for air targets.

Some variants of solving the problem of ISAR imaging in case of non-uniform rotation of the object around single axis were proposed in [5, 18]. However, for more complex conditions of target flight in a turbulent atmosphere the decision hasn’t been received. Irregular spatial movement of the
object, which can be modeled in decimeter-centimeter range of radio wave lengths by a set of spatially distributed scatterers [19], significantly complicates the problem of radar imaging. To solve it, one needs to restore the spatial distribution of the scatterers. Nowadays, the problem of 2D imaging under the influence of destabilizing factors is a very important one. The most significant of these factors is the influence of random spatial movement of the object. Restoration of ISAR image in this case requires: first, alignment of HRRP by range along the LOS; second elimination of the of random phase influence due to translational motion of the target, instabilities of receiver and transmitter; and, finally, third, elimination of random phase components of signals echoed from individual scatterers due to irregular rotational movement of target to adapt to the random movement component.

One of the major problems in ISAR imaging is that the image obtained at the end of this process is a 2D projection of the true target reflectivity pattern onto the ISAR Image Projection Plane (IPP) [20]. To circumvent these issues, one usually assumes that data is to be collected over a long time interval in order to get at least one suitable frame with desirable resolution and IPP [21].

In [22, 23], the coincidence 3D imaging method in multi-site and one-site radars was proposed. In this method, the target centers of scattering were resolved in the plane perpendicular to the LOS by means of measuring the phases of their independent echo waveforms. This method, as opposed to conventional radar imaging methods based on the Doppler principle, the radar image can be obtained both for stationary targets and maneuvering ones. But to achieve that, it is necessary that a single scatterer be present in every individual range cell. Under the latter condition, this 3D imaging method has the potential advantages. Otherwise, the 3D imaging will be impossible for such range cells. For one-site radar, this imaging technique has an issue with correctly recovering the scale of image if the target movement has random component. The possibility of using the phase method was also discussed in [5] to estimate the scatterer azimuth in multi-site radar, and in [24] the same possibility was discussed as applied to one-site radar, where imaging employed multiple antennas put at different heights. Then, the relative height of scatterers could be estimated by measuring the phase differences between the two or more 2D images.

The traditional approach to spectral estimation is to use nonparametric methods, such as Fourier transform, to evaluate the spectrum. Modern spectral-estimation techniques based on parametric modeling of the signal time behavior, were used in [5, 25] for ISAR imaging capable of better cross-range resolution.

In [13, 26], it was shown that the presence of random components of the spatial movement lead to different phases changes of signals from different scatterers, which are not linear. Therefore, to obtain the fully focused image of the target given small angles of target turn with respect to radar it is not enough to have two scatterers, one of which being used as the reference, and the phase change of the other being used to restore the object yaw [5]. In [26], an adaptive imaging algorithm was proposed for the object that involved autofocus (selection of phase changes of different scatterers without using information about their cross-range). However, the imaging result is largely dependent on autofocus criterion selection. The incorrectness of the problem actually leads to an ambiguous solution.

So, the aim of the article is to develop the 2D ISAR method, which takes into account the random spatial components of the rotational movement along with the regular component of target movement.

**Main part**

Vector of the echo signal complex envelope at the output of linear section of the receiver can be calculated as follows [5]

\[
X(t) = \sum_{i=1}^{I} A_i(r^0) S_i U(t-2\rho_i r^0/c) e^{-j4\pi\rho r^0},
\]

where \(A_i(r^0)\) is the polarization scattering matrix of \(i\)-th scatterer; \(S_i\) is the polarization vector of incident wave; \(I\) is the number of illuminated local scatterers; \(r^0\) is the unit vector of the incident wave; \(U(t)\) is the complex envelope of signal at the output of optimal processing device; \(\rho_i\) is the radius vector of the \(i\)-th scatterer phase center; \(\nu^T\) is the symbol of transposition; \(c\) is speed of light.

Movement of scatterers (“bright points”) associated with a change in spatial position of targets with respect to the radar while the observation is in progress. We show that in contrast to the 2D movement, the spatial (or 3D) one leads to different phase dependencies for different scatterers. In this case, considering the rotation matrix given small angles of yaw \(\psi\), roll \(\gamma\), pitch \(\theta\), and elevation \(\varepsilon\) of target being observed at the azimuth \(\beta\) ratio, the phase multiplier entering expression (1) for the \(i\)-th scatterer with coordinates \((x_i, y_i, z_i)\) can be approximately computed, given that the right-hand Cartesian coordinate system has an origin at the center of the object’s rotation.
\((x'_r, y'_r, z'_r)\), as follows:

\[ p^r_{\text{rr}} = \left( x' - x'_r \right) (\cos \beta - \psi \sin \beta) + \left( z' - z'_r \right) (\psi \cos \beta + \sin \beta) + \left( y' - y'_r \right) (\gamma \sin \beta - \theta \cos \beta + \varepsilon). \tag{2} \]

Small changes of angles \(\psi, \gamma\), and \(\theta\) are typical, for example, given straight-line flight of air target in the turbulent atmosphere. Initial elevation angle of ground, surface, and air targets given observation of the latter at long-ranges is usually small. Besides, the coordinates of the rotation center entering equation (2), \((x'_r, y'_r, z'_r)\), are set by the dominant scatterer algorithm (DSA) [11] or multiple scatterer algorithm [5]. Initial orientation of the axes of coordinate system \(o'x'y'z'\) at the beginning of target observation, when \(\psi = \theta = \gamma = 0\), is as follows: axis \(o'x'\) is directed forward, axis \(o'z'\) is directed toward the right wing and axis \(o'y'\) goes straight up (axes \(o'x', o'y', o'z'\) at this point are parallel to the axes \(o_x, o_y, o_z\) of ground coordinate system centered at the point of radar standing). This coordinate system is used to describe the object position when it makes consecutive turns by angles of roll \(\gamma\), pitch \(\theta\), and yaw \(\psi\) [5]. While deriving the (2), we assumed that target flew straight forwardly along the axis \(o_x\), azimuth angle \(\beta\) being counted off clockwise from that axis in the plane \(o_xo_y\).

Since the three components in (2) have different weight depending on the position of the scatterer at the object, then random changes of orientation angles do not equally affect the range shifts of different scatterers with respect to the radar. Dependences of signal phases from these scatterers in general will be different and will not be linear. Random variations of the target spatial orientation (changes in angles of roll and pitch) lead to different random dependences of signal phase from each bright point of target. Use of spectral analysis methods will not be optimal for imaging. If the target observation interval is divided into relatively short subintervals, during which target moves almost straight, these methods can be employed at such intervals. However, reduced time of target observation will worsen the resolution across LOS. Improvement of the restored image in such cases is possible by measuring the phase dependences of several "bright points" in a sequence of complex ISAR images of target and accounting for these phase changes for each point while obtaining the final image. To better measure the position of bright points at the target, they are being tracked by parametric spectral analysis methods used for getting a sequence of images and then these results are further used for measuring the phase dependences of selected bright points at the radar image obtained by using the discrete Fourier transform (DFT).

Along with classic DFT, the parametric Capon \(f_k(f)\) and maximum entropy \(f_{\text{ME}}(f)\) methods were used for imaging according to [5, 26]

\[ f_k(f) = \frac{1}{|V(f)^T A^{-1} V(f)|}, \tag{3} \]

\[ f_{\text{ME}}(f) = \frac{g^T A^{-1} g}{|g^T A^{-1} V(f)|^2}, \tag{4} \]

where \(f\) is a signal spectrum frequency; \(V(f)\) is complex non-random \(N\)-dimensional frequency length, which depends on the frequency \(f\); \(g^T = [1, 0, 0, ..., 0]\) is the \(N\)-dimensional vector of coefficients; \(A^{-1}\) is the inverse correlation matrix of observation.

Practically, functions (3) and (4) are formed by scanning the search vector \(V(f)\) over some frequency range while replacing the true correlation matrix of input signal \(A\) by its maximum likelihood estimate

\[ \hat{A} = A + \Delta A, \tag{5} \]

where \(\Delta A\) is the matrix of internal system disturbances.

Given uniform rectilinear motion of targets, the phases of echo signals from bright points change linearly, and the target image may be recovered by observing the target along its entire trajectory (Fig. 1).

The simulation results were obtained for B-52 aircraft under the following conditions: flight altitude – 8000 m, velocity – 750 km/h, target distance – 100 km, initial course angle of target observation – 89º (Fig. 1), observation interval was 8 s given the flight without atmospheric disturbances. Radar specifications: wavelength – 3 cm, linearly-frequency modulated probing pulse with rectangular envelop (pulswidth was 13.65 μs, frequency deviation was 150 MHz). Simulation was performed using the program RTBS [27] according to mathematical model given in [3] and supplemented with image recovery block (in MATLAB).

Output data about projections of “bright points” onto the line of sight along with their cross-sections were transferred from the RTBS software to the
MathCad, where the 1024 complex HRRP were formed for the target observation interval (Fig. 2).

Target motion under condition of atmospheric disturbances (random changes in angles $\psi$, $\gamma$ and $\theta$) led to phase modulation of signal from each "bright point" and to the image distortion (Fig. 3).

Fig. 4 shows examples of images obtained during single subintervals of 1 s duration. Despite degraded cross-range resolution, it is possible to measure the coordinates of individual peaks at the images. The five bright points numbered from 1 through 5 marked out at the aircraft image (Fig. 1, b) were selected for investigation.

Point number 0 was used as a reference in the DSA. For each of the selected “bright point”, the changes in their phase were measured given rectilinear target’s movement with no influence (Fig. 5) and under the influence of atmospheric turbulence upon the target flight (Fig. 6).
Figure 4 – Two-dimensional image of B-52 aircraft obtained during short sections of flight path: a) obtained by FFT, b) obtained by the method of maximum entropy, c) obtained by the Capon method.

Figure 5 – Recovered phase dependences of bright points under condition of ideal rectilinear motion of target given only DFT was used (a), and given a combination of the maximum entropy method and DFT was used (b).

Figure 6 – Recovered phase dependences of bright points under condition of rectilinear target flight given practically quiet atmosphere when only DFT was used (a), and when a combination of the maximum entropy method and DFT was used (b).
In case of uniform rectilinear motion, the phases of all points at target’s image vary linearly (Fig. 5). Moreover, the use of maximum entropy method for measuring the scatterer positions within the observation window, in some cases, significantly improves recovering of the phase history (as for the point #1) (taking into account the ambiguity of phase measurements). In case of the target rectilinear motion under the influence of atmospheric disturbances, the phase history at each DFT spectrum point is nonlinear due to random changes of target orientation in space (Fig. 6).

Notice that, as seen on the set of HRRP (Fig. 2), the bright point number 1 almost disappears starting with profile #800 (shaded by target elements), which is displayed at the graph of its phase. Phases of all points to the left from the center of rotation (point «0», Fig. 5) change toward negative values and phase of points to the right of that center change toward positive values.

To restore the image, we proposed to select the sequence of complex HRRP for each of the chosen bright points in such manner that the phase of signals from these points would vary linearly. Such dependencies shall correspond to the regular component of target movement (Fig. 5). This can be done using the least squares method and providing linear approximation for every chosen point (Fig. 7). Then, selection of such HRRP must be carried out, for which the signal phase change at corresponding point in the Fourier spectrum is approaching this line (Fig. 7). Since the number of HRRP gets smaller (from 896 to 512) this straight line at the graph has a greater slope (Fig. 7).

Then, the phase variation of each selected point is reduced to a straight line. So, the point would appear focused better at the image. Number of received two-dimensional radar images will match the number of selected bright points. Having obtained all individual images, we combine them into the whole one. In this case, parts of the image selected in this manner appear focused better at the final image. The most effective approach is to choose the cross-range bands along the line of sight. Imaging results are shown in Fig. 8.

![Figure 7](image7.png)  
**Figure 7** – Recovered phase change of a bright point: data1 – raw phase change, data2 – approximating curve, data3 – aligned phase change

![Figure 8](image8.png)  
**Figure 8** – Two-dimensional image of target: during the flight without atmospheric disturbances (a), during the flight in turbulence quiet atmosphere (b) and restored target image (c)

**Conclusions**

Improved method of two-dimensional imaging of air targets during its rectilinear flight in conditions of atmospheric turbulence, which
involves tracking of the scatterers position at the partial images given reduced subintervals of observation, was proposed. For this purpose, we proposed to use parametric methods of spectral analysis.

The phase dependences for these points measured by accumulation of radar images, obtained by DFT, were used to restore several images with focus around the selected point. These images then combined into a single one.

Example of simulation results of two-dimensional imaging of B-52 aircraft model, which illustrate the theoretical possibility of such an approach, was presented.

Further directions for improving this method is to consider objects as three-dimensional and use the possibility of measuring the third coordinate by phase method and also to look for capabilities of reducing the computation burden for its implementation.

References


МЕТОД ВОЗСТАНОВЛЕНИЯ РАДИОИЗОБРАЖЕНИЙ ОБЪЕКТОВ С ПРОСТРАНСВЕННЫМ ВРАЩАТЕЛЬНЫМ ДВИЖЕНИЕМ В РЛС С ИНВЕРСНЫМ СИНТЕЗИРОВАНИЕМ АПЕРТУРЫ

Исследуется влияние случайных составляющих пространственного движения на законы изменения фаз сигналов от различных отражателей. Разработан метод восстановления радиоизображения объекта, который наряду с регулярной составляющей траекторного перемещения учитывает случайные пространственные составляющие вращательного движения. Эти составляющие оцениваются по изменениям фаз блестящих точек на последовательности радиоизображений, полученных с использованием дискретного преобразования Фурье, координаты точек оцениваются с использованием параметрических методов спектрального анализа.

Ключевые слова: радиолокационный дальностный портрет, радиоизображение, инверсное синтезирование апертуры, поперечная дальность, пространственное вращательное движение.